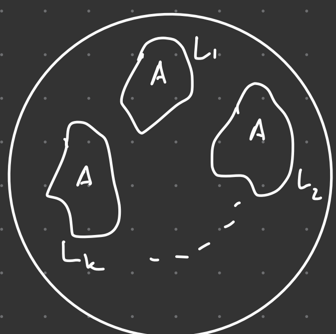
Hofer Pseudonorms on Braid Groups

(M,ω) symplectic mfd $\Rightarrow$ $\mathrm{Ham}(M,\omega) = \{\phi \in \mathrm{Diff}(M) \mid \phi = \phi^1_H,\ \exists H \in \mathcal{C}^\infty(M \times S^1)\}$

$\|\cdot\|_H : \mathrm{Ham}(M,\omega) \to \mathbb{R}_{\geq 0}$ Hofer norm ($\|\phi\|_H :=$ "minimal energy of an isotopy $Id \rightsquigarrow \phi$)

$\Rightarrow$ difficult to bound it from below.



$(M,\omega) = \mathbb{D}$, $L_i \cong S^1$ $i = 1, \dots, k$

$A \in \left(\frac{1}{k+1}, \frac{1}{k}\right)$

$\phi \in \mathrm{Ham}(\mathbb{D}) \mid \exists \sigma \in \mathfrak{S}_k$,

$\phi(L_i) = L_{\sigma(i)}\ \forall i$

$\Rightarrow b(\phi, \underline{L}) \in \mathcal{B}_k$ is well defined

(Frédéric Le Roux): $\|g\|_{\underline{L}} = \inf_{b(\phi,\underline{L}) = g} \|\phi\|_H$

Q: Non-trivial lower bounds for $\|\cdot\|_{\underline{L}} : \mathcal{B}_k \to \mathbb{R}_{\geq 0}$?

Theorem $\|\phi\|_H \geq \frac{1}{4k} \frac{(k+1)A - 1}{4k(k-1)} |\ell k\, b(\phi, \underline{L})|$

Proof: Spectral invariants from Link Floer Homology.

Corollary $\|g\|_{\underline{L}} \geq O(|\ell k(g)|)$